Intermittency and scaling laws for wall bounded turbulence

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Abstract

Well defined scaling laws clearly appear in wall bounded turbulence, even very close to the wall, where a distinct violation of the refined Kolmogorov similarity hypothesis (RKSH) occurs together with the simultaneous persistence of scaling laws. A new form of RKSH for the wall region is here proposed in terms of the structure functions of order two which, in physical terms, confirms the prevailing role of the momentum transfer towards the wall in the near wall dynamics.

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The intermittent behavior of velocity increments in the inertial range of fully developed turbulence has been a subject of renewed interest during the years, starting from the objection that Landau raised to Kolmogorov theory of 1941 (K41). Since then, any theory of the inertial range can not avoid considering the effect of intermittent dissipation of energy on the inertial scales of motion. Under this respect, the Kolmogorov-Obukhov refined similarity hypothesis (RKSH), certainly the most credited[6], leads to a probability distribution function of velocity increments characterized by the scaling

$$< \delta V^p > \; \propto < \epsilon_r^{p/3} > r^{p/3} \,, \tag{1}$$

where ϵ_r^q denotes the q^{th} moment of the dissipation spatially averaged over a volume of characteristic dimension r and the brackets indicate ensemble averaging. Taking into account the scaling properties of the dissipation field,

$$< \epsilon_r^q > \; \propto r^{\tau(q)} \,, \tag{2}$$

equation (1) implies that the velocity structure function of order p is expressed as a power law of the separation with exponent

$$\zeta_p = \tau(p/3) \, + \, p/3 \,. \tag{3}$$

Here, the anomalous correction, $\tau(p/3)$, to the K41-exponent accounts for the intermittency of the velocity increments in the inertial range of homogeneous and isotropic turbulence.

A substantial extension of the range of scales where similarity is observed has recently been achieved[4] by assuming, as basic quantity, the third order structure function instead of the separation r,

$$< \delta V^p > \; \propto \frac{< \epsilon_r^{p/3} >}{< \epsilon >^{p/3}} < \delta V^3 >^{p/3} \,, \tag{4}$$

as suggested by the Kolmogorov equation[6]. A direct consequence of eq. (4) is the existence of an extended self-similarity (ESS) of the generic structure function of order p in terms of the third order moment with exponent ζ_p. Since its introduction, the generalized Kolmogorov similarity hypothesis (4) has appeared as the characteristic feature of a vast number of turbulent systems.

In the present letter we intend to discuss the issue of intermittency in wall bounded turbulence and its relationship with scaling (ESS) laws, which have been observed[1],[2] even in regions very close to the wall dominated by quite ordered vortical structures[5]. As shown in fig. (1), we have evidence that intermittency increases moving from the bulk of the fluid towards the wall[2]. In principle, one may attempt to describe this behavior in the framework of RKSH, in its generalized form (4). Hence the larger intermittency (smaller ζ_p) would be provided by an increase of intermittent fluctuations of ϵ_r (larger values of $|\tau(p)|$). In such conditions, the anomaly of the scaling exponents would strongly depend on the local flow properties, loosing, thus, any trait of universality.

To assess the self-consistency of this approach, in fig (2) we plot on a logarithmic scale the structure function of order six versus $< \epsilon^2 >< \delta V^3 >^2$. On the basis of the assumed validity of (4), the plot should result in a straight line of slope $s = 1$, independent of the distance from the wall. This behavior actually emerges near the center of the channel while in the wall region a quite clear, though small, violation is manifested. Specifically, for $y^+ = 31$ two different scaling laws appear. The one, characterized by slope $s = 1$, trivially pertains to the dissipative range. The other, with slope $s = .88$, which doesn't satisfy (4), shows a first clear example of failure of RKSH.

The previous discussion may suggest a relationship between the increase of intermittency, observed in the near wall region, and the simultaneous breaking of the RKSH. To this regard, it seems interesting to investigate the possible existence of a new form of RKSH valid in the near wall region. In fact RKSH, somehow suggested by the well known "4/5" Kolmogorov equation (see Frish[6]), tells us, in physical terms, that the "energy flux" in the inertial range, represented by the term $(\delta V_r)^3$, fluctuates with a probability distribution which is the same of ϵ_r. However, in the case of strong shear, we should expect that a new term, proportional to $\partial_z < U > (\delta V_r)^2$, enters the estimate of the energy flux at scale r. Such a new term, indeed, appears in the analysis performed for homogeneous

shear flows (see for instance Hinze[7]). If this term becomes dominant, as it may occur for a very large shear, one is led to assume that the fluctuations of the energy flux in the inertial range are proportional to $(\delta V_r)^2$, *i.e.* $\epsilon_r \propto A(r)(\delta V_r)^2$, with $A(r)$ a non fluctuating function of r. Hence, we may expect that a new form of the RKSH should hold which, in its generalized form, reads as

$$< \delta V^p > \; \propto \frac{< \epsilon_r^{p/2} >}{< \epsilon >^{p/2}} < \delta V^2 >^{p/2} \; . \tag{5}$$

The above expression of the new RKSH is given in terms of the structure function of order two, without explicit reference to the separation r, in the same way as the generalized RKSH (4). In the spirit of the extended self similarity, we assume the new form of RKSH to be valid also in the region very close to the wall, where the shear is certainly prevailing.

In order to verify this set of assumptions, we show in fig. (3) a log-log plot of equation (5) for $p = 4$ at $y^+ = 31$. In the insert, we show for the same plane the compensated plot of both (5) for $p = 4$ and (4) for $p = 6$. It follows a quite clear agreement of eq. (5) with the numerical data. In principle, the function $A(r)$ might be evaluated theoretically starting from the Kolmogorov equation for anisotropic shear flow (e.g. see[3]).

The increased intermittency of the velocity fluctuations near the wall may be estimated by considering how the flatness $F(r)$ grows with $r \to 0$, with

$$F(r) = \frac{< \delta V^4(r) >}{< \delta V^2(r) >^2} \; . \tag{6}$$

By combining the definition (6) with (4) and (5) we obtain the following expressions in terms of ϵ_r,

$$F_b \; = \frac{< \epsilon_r^{4/3} >}{< \epsilon_r^{2/3} >^2} \qquad\qquad F_w \; = \frac{< \epsilon_r^{4/2} >}{< \epsilon_r^{2/2} >^2} \, , \tag{7}$$

which are suitable for the bulk and near the wall region, respectively. As we see from fig. (4), both F_b and F_w diverge for $r \to 0$, indicating intermittent behavior in both cases, if we exclude the smallest separations falling into the dissipative range. Clearly F_w diverges faster than F_b. This result is consistent with the corresponding analysis performed directly in terms of structure functions of velocity by means of eq. (6) and provides a further evidence of the validity of (5) near the wall. In

fact, the application of F_b near the wall doesn't catch the increase of intermittency of the velocity fluctuations (see fig. (4)). On the other hand, the differences in the statistical properties of the dissipation between the bulk and the near wall region are too small to account for the increase of intermittency of the velocity increments near the wall. This is indirectly confirmed by the observed direct scaling (ESS) of the structure functions with $< \delta V^3 >$, which implies, starting from eq. (5),

$$\hat{\tau}(p/2) = \hat{\zeta}_p - \frac{p}{2}\hat{\zeta}_2 , \tag{8}$$

where a hat has been introduced here to denote the scaling exponents with respect to $< \delta V^3 >$. This distinction was not necessary in the bulk region where $\tau \equiv \hat{\tau}$. By using expression (8) near the wall and eq. (3) in the bulk region we obtain that the "intermittency correction" $\hat{\tau}(q)$ results to be essentially independent of the distance from the wall, fig. (5). Hence the observed increase of intermittency of the velocity increments seems to be associated more to the structure of the RKSH rather than to the intermittency of dissipation. These theoretical findings seem to be confirmed by experimental results in a flat plate boundary layer obtained recently by Ciliberto and coworkers (private communication).

We like here to emphasize that, to verify the new RKSH, we selected on purpose the plane closest to the wall where scaling laws still appear. On the opposite, in the bulk region, the original RKSH holds. At intermediate planes we expect the scaling exponents to emerge from a complex blending of these two basic behaviors, leading to a continuous variation with the distance from the wall[2].

In conclusion, we have found that a quite evident failure of the RKSH occurs in the near wall turbulence in correspondence with the simultaneous appearance of scaling laws. The new form of the RKSH we have proposed in this letter for the wall region is expressed in terms of the structure function of order two, instead of the structure function of order three as in the original form. This may be seen as a statistical representation of the physical features of the near wall region, which is controlled more by the mechanism of momentum transfer rather than by the classical energy cascade.

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FIGURES

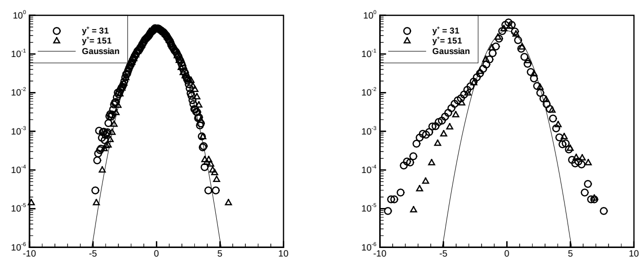

FIG. 1. Pdf of the velocity increments for different values of the separation (left $r^+ = 160$, right $r^+ = 18$) at two distances from the wall: $y^+ = 151$, near the center of the channel, and $y^+ = 31$, in the wall region of the flow. Data from DNS of a turbulent channel flow with $Re_* = 160$[1]. Wall units are used throughout the paper.

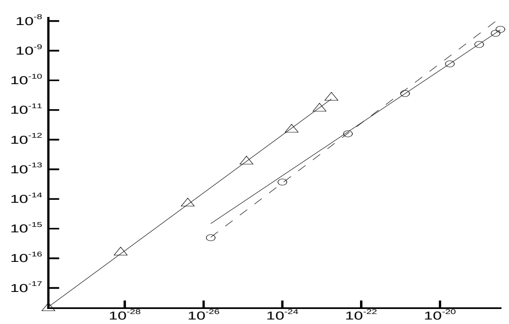

FIG. 2. $< \delta V^6 >$ vs $< \epsilon^2 >< \delta V^3 >^2$ for two different wall normal distances. Bulk region ($y^+ = 151$): data (triangles) and their fit in the region $r^+ \in [20, 320]$ (solid line with slope .99). Wall region ($y^+ = 31$): data (circles) and their fits in the two regions $r^+ \in [1, 20]$ and $r^+ \in [20, 320]$, solid line with slope .99 and dotted line with slope 0.88, respectively.

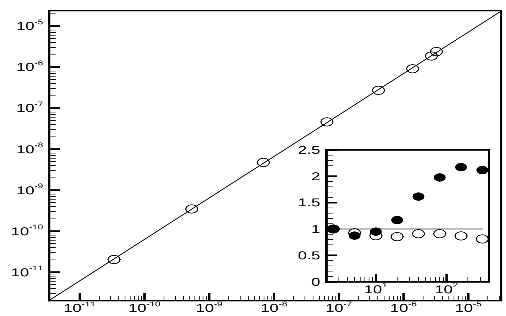

FIG. 3. Check of consistency for eq. (5) at $y^+ = 31$: $< \delta V^4 >$ vs $< \epsilon^2 >< \delta V^2 >^2$. The solid line (slope 1.01) gives the fit in the whole range. In the insert: open circles, compensated plot for eq. (5), $< \delta V^4 > / < \epsilon^2 >< \delta V^2 >^2$ vs r^+; filled circles, corresponding plot for eq. (4), $< \delta V^6 > / < \epsilon^2 >< \delta V^3 >^2$ vs r^+.

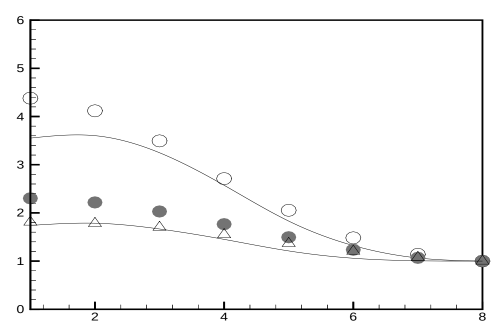

FIG. 4. Flatness, $F =< \delta V^4 > / < \delta V^2 >^2)$ vs $\log_2(r^+/Dx^+)$, $Dx^+ = 2.5$, at $y^+ = 151$ (open triangles) and $y^+ = 31$ (open circles), as evaluated by eqs. (7), using F_b and F_w, respectively. For comparison: filled circles, F_b applied at $y^+ = 31$. Correspondingly, the solid lines give the flatness as evaluated directly in terms of velocity.

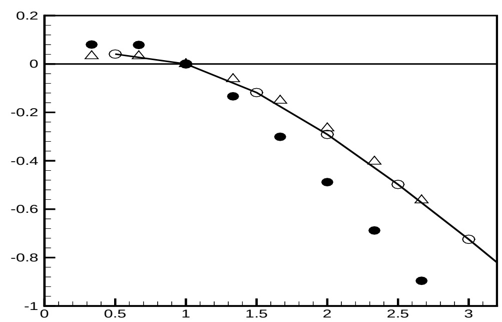

FIG. 5. The anomalous correction $\hat{\tau}(q)$ as computed by the new scaling law for the wall region ($y^+ = 31$), eq. (5), (open circles) compared with that issuing from RKSH at both $y^+ = 151$ (triangles) and $y^+ = 31$ (filled circles).